Cardinality-Regularized Hawkes-Granger Model

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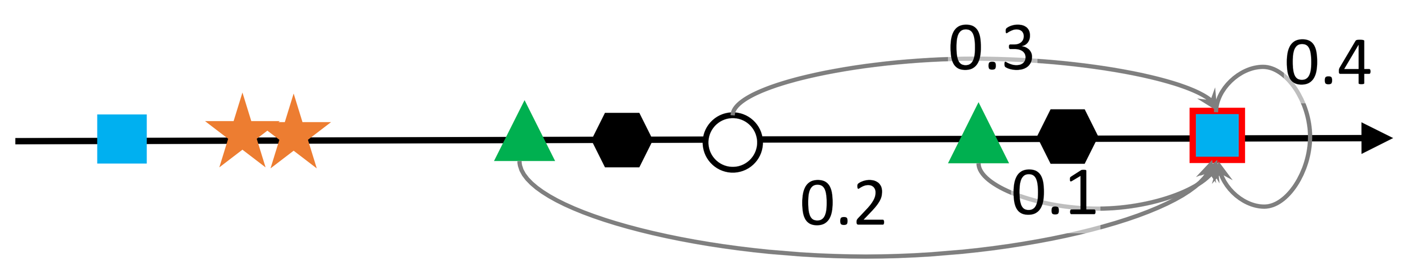
Problem: Event causal analysis

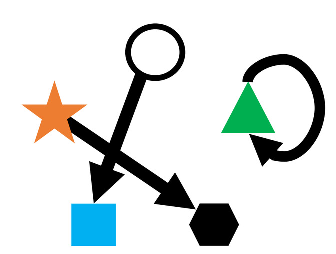
- Given: marked temporal events,

$$\mathcal{D} = \{(t_0, d_0), (t_1, d_1), \dots, (t_N, d_N)\}$$

- Answer "who caused this?"
  - Instance-level triggering probabilities (for each instance)



  - Type-level causal graph



Model: Hawkes process

- Basic quantity: intensity function

$$\lambda_d(t \mid \mathcal{H}) = \mu_d + \sum_{i:t_i<t} A_{d,d_i} \phi_d(t - t_i)$$

  - Probability of having the first type-*d* event after observing the event history $\mathcal{H}$.
  - μ_d : baseline intensity of type-d events
  - $\phi_d(t)$: decay function of the type-d
  - $A_{k,l}$: Impact matrix (btwn. type k and l)

- Relationship with Granger causality
  - If $A_{k,l} = 0$, then type l is Granger-non-cause of type k

Challenges in maximum likelihood

- Log likelihood

$$L = \sum_{n=1}^{N} \left\{ \ln \lambda_{d_n}(t_n \mid \mathcal{H}_{n-1}) - \int_{t_{n-1}}^{t_n} du\ \lambda_{d_n}(u \mid \mathcal{H}_{n-1}) \right\}$$

- Native maxim likelihood is numerically very tricky
- EM-like algorithm ("MM") is efficient but does not lead to a sparse solution

What about existing "sparse" Hawkes models?

Theorem 1: The L1-regularized Hawkes model does not provide any sparse solution for **A**.

- Simply because the Jensen bound of log-likelihood depends on $+\ln A_{k,l}$
  - Prohibits $A_{k,l} = 0$ for all (k, l).

Research question

How do we get a sparse solution in the MM framework of the Hawkes process?

L0-regularized Hawkes process

- Log likelihood for **A** under L0 (and L2) penalty takes the form of

$$\sum_{k=1}^{D} \sum_{l=1}^{D} \left(Q_{k,l} \ln A_{k,l} - H_{k,l} A_{k,l} - \frac{1}{2} \nu_A {A_{k,l}}^2 \right) - \tau \|\mathsf{A}\|_0$$

Main contribution

- Found a semi-analytic solution under L0 penalty with the notion of "ϵ-sparsity"
- Applied it to simultaneous instance- and type-level sparse causal diagnosis

Instance- and type-level Granger causal diagnosis: How?

- Type-level causal analysis:
  - Look at $\{A_{k,l}\}$ – If 0, no causal relation btwn k,l.
- Instance-level causal analysis:
  - Look at the variational distribution $\{q_{n,i}\}$ in the MM algorithm – degree of how strongly instance n was triggered by instance i

Applications (→paper)

- Event grouping and de-duplication in AIOps
- Failure propagation analysis in power grid